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NHDS: The New Hampshire Dispersion relation Solver

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In collisionless astrophysical plasmas, waves and instabilities are well modeled by the linearized Vlasov–Maxwell equations, which have non-trivial solutions only when the complex frequency ω solves the hot-plasma dispersion relation. *NHDS* (New Hampshire Dispersion relation Solver) is a numerical tool written in Fortran 90 and first introduced by Verscharen et al. (2013) to solve this dispersion relation under the assumption that the plasma background distribution is a gyrotropic drifting bi-Maxwellian for each species j,

$$f_{0j}(v_\perp, v_\parallel) = \frac{n_j}{\pi^{3/2} w_{\perp j}^2 w_\parallel} \exp\left(-\frac{v_\perp^2}{w_{\perp j}^2} - \frac{\left(v_\parallel - U_j\right)^2}{w_{\parallel j}^2}\right), \tag{1}$$

in a cylindrical coordinate system aligned with the direction of the background magnetic field $\boldsymbol{B}_0$, where n_j is the density, $w_\perp$ ($w_\parallel$) is the perpendicular (parallel) thermal speed with respect to $\boldsymbol{B}_0$, and U_j is the field-aligned drift speed. All floating-point quantities use double precision.

The *NHDS* code closely follows the formulation of the hot-plasma dispersion relation laid out by Stix (1992). It uses a Newton-secant method to identify those frequencies at which there are non-trivial solutions to the wave equation,

$$\begin{pmatrix} \epsilon_{xx} - \dfrac{k_z^2 c^2}{\omega^2} & \epsilon_{xy} & \epsilon_{xz} + \dfrac{k_\perp k_z c^2}{\omega^2} \\ \epsilon_{yx} & \epsilon_{yy} - \dfrac{k^2 c^2}{\omega^2} & \epsilon_{yz} \\ \epsilon_{zx} + \dfrac{k_\perp k_z c^2}{\omega^2} & \epsilon_{zy} & \epsilon_{zz} - \dfrac{k_\perp^2 c^2}{\omega^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0, \tag{2}$$

based on an initial guess for ω, where ϵ is the dielectric tensor, $\boldsymbol{E}$ is the vector of the electric-field Fourier amplitudes, c is the speed of light, and $\boldsymbol{k} = (k_\perp, 0, k_z)$ is the wavevector. The initial guess defines the plasma mode that the code follows in $\boldsymbol{k}$. The Newton-secant method converges if the absolute value of the determinant of the matrix in Equation (2) is less than a user-defined value. All frequencies are given in units of the proton gyro-frequency Ω_p and all length scales in units of the proton inertial length d_p.

For each of the up to ten plasma components j, the user defines the temperature anisotropy $T_{\perp j}/T_{\parallel j}$ with respect to $\boldsymbol{B}_0$, the value of $\beta_{\parallel j} \equiv 8\pi n_j k_\mathrm{B} T_{\parallel j}/B_0^2$, the relative charge q_j/q_p, the relative mass m_j/m_p, the relative density n_j/n_p, and the normalized drift velocity U_j/v_A, where k_B is the Boltzmann constant, v_A is the proton Alfvén speed, and $T_{\parallel j}$ is the temperature parallel to $\boldsymbol{B}_0$. Furthermore, the ratio v_A/c and the angle of propagation θ are user-defined parameters.

The calculation of ϵ_{ik} entails the evaluation of the modified Bessel function $I_m(\lambda_j)$ of the first kind and the plasma dispersion function $Z(\zeta)$, where $\lambda_j \equiv k_\perp^2 w_{\perp j}^2/2\Omega_j^2$, and ζ is a dimensionless complex number. For the evaluation of I_m, *NHDS* applies the recursion method supplied by the *Numath* Library (Clenshaw 1962). It determines the maximum order m_max of I_m as the smaller of either a user-defined limit or as the number for which $I_{m_\mathrm{max}}(\lambda_j)$ is less

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than a user-defined value. *NHDS* evaluates $Z(\zeta)$ following Poppe & Wijers (1990) by computing the complex error function $w(\zeta) = Z(\zeta)/i\sqrt{\pi}$ through one of the following methods, depending on the value of $|\zeta|$: a power series, the Laplace continued fraction method, or a truncated Taylor expansion. This combined method is faster than alternative approaches and calculates $w(\zeta)$ to an accuracy of 14 significant digits for almost all ζ.

NHDS determines the polarization of the wave solutions as the ratios E_y/E_x and E_z/E_x from Equation (2), which translate to ratios of the magnetic-field amplitudes through Faraday's law. In addition, as described by Verscharen & Chandran (2013) and Verscharen et al. (2016), *NHDS* calculates the relative wave energy W_k and the Fourier amplitudes of the fluctuations in density, bulk velocity, and pressure. The code also calculates the contribution γ_j to the total growth/damping rate $\mathrm{Im}(\omega)$ from each species j as described by Quataert (1998).

For a given wave solution, *NHDS* can determine the value of the self-consistent fluctuating distribution function on a user-defined Cartesian grid in velocity space as described by Verscharen et al. (2016). *NHDS* saves the fluctuating distribution function in *HDF5* files and creates an *XDMF* file for visualization with programs like *ParaView*. This calculation entails the calculation of the Bessel function $J_m(k_\perp v_\perp/\Omega_j)$ of order m, which *NHDS* performs through a polynomial Chebyshev approximation. The maximum order $m_{\max}$ for J_m is determined in the same way as $m_{\max}$ for I_m in the calculation of ϵ_{ik}, except that $m_{\max}$ for J_m is evaluated for each $v_\perp$.

Figure 1 shows the dispersion relations of Alfvén/ion-cyclotron (A/IC) and fast-magnetosonic/whistler (FM/W) waves in parallel and perpendicular propagation as well as some of their polarization properties determined with *NHDS*.

The code is publicly available for download (Verscharen & Chandran 2018, Codebase: https://github.com/danielver02/NHDS).

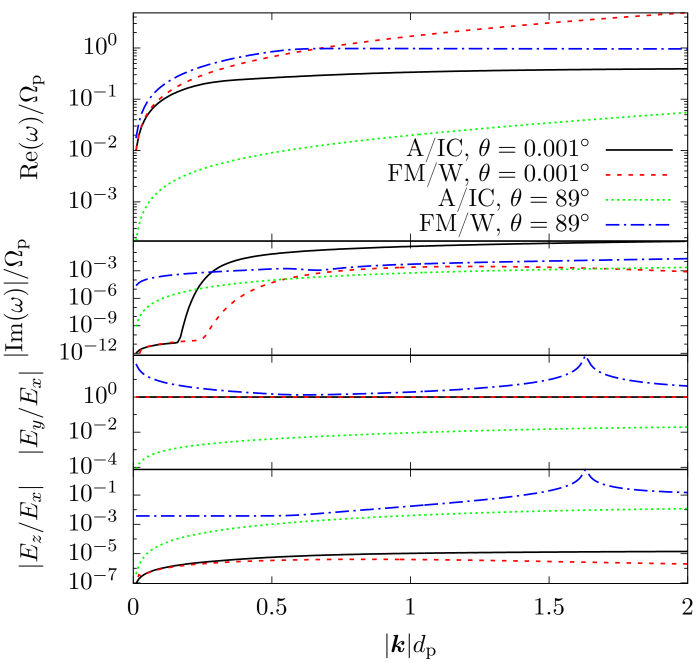

Figure 1. Dispersion relations for the A/IC and FM/W waves in parallel ($\theta = 0.001°$) and perpendicular ($\theta = 89°$) propagation. The panels show from the top to the bottom: the normalized real part of the frequency, the normalized damping rate, the ratio $|E_y/E_x|$, and the ratio $|E_z/E_x|$ as functions of $|\boldsymbol{k}|$.

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